

Problem 1.52

[Difficulty: 4]

1.52 An American golf ball is described in Problem 1.42. Assuming the measured mass and its uncertainty as given, determine the precision to which the diameter of the ball must be measured so the density of the ball may be estimated within an uncertainty of ± 1 percent.

Given: American golf ball, $m = 1.62 \pm 0.01$ oz, $D = 1.68$ in.

Find: Precision to which D must be measured to estimate density within uncertainty of ± 1 percent.

Solution: Apply uncertainty concepts

Definition: Density, $\rho \equiv \frac{m}{V} \quad V = \frac{4}{3} \pi R^3 = \frac{\pi D^3}{6}$

Computing equation:
$$u_R = \pm \left[\left(\frac{x_1}{R} \frac{\partial R}{\partial x_1} u_{x_1} \right)^2 + \cdots \right]^{\frac{1}{2}}$$

From the definition,

$$\rho = \frac{m}{\pi D^{3/6}} = \frac{6m}{\pi D^3} = \rho(m, D)$$

Thus $\frac{m}{\rho} \frac{\partial \rho}{\partial m} = 1$ and $\frac{D}{\rho} \frac{\partial \rho}{\partial D} = 3$, so

$$u_\rho = \pm [(1 u_m)^2 + (3 u_D)^2]^{\frac{1}{2}}$$

$$u_\rho^2 = u_m^2 + 9 u_D^2$$

Solving,

$$u_D = \pm \frac{1}{3} [u_\rho^2 - u_m^2]^{\frac{1}{2}}$$

From the data given,

$$u_\rho = \pm 0.0100$$

$$u_m = \frac{\pm 0.01 \text{ oz}}{1.62 \text{ oz}} = \pm 0.00617$$

$$u_D = \pm \frac{1}{3} [(0.0100)^2 - (0.00617)^2]^{\frac{1}{2}} = \pm 0.00262 \text{ or } \pm 0.262\%$$

Since $u_D = \pm \frac{\delta D}{D}$, then

$$\delta D = \pm D u_D = \pm 1.68 \text{ in.} \cdot 0.00262 = \pm 0.00441 \text{ in.}$$

The ball diameter must be measured to a precision of ± 0.00441 in. (± 0.112 mm) or better to estimate density within ± 1 percent. A micrometer or caliper could be used.